INTRODUCTION TO QUANTUM MECHANICS - I

From the failure of classical physics to the Schrödinger equation. We cover wave–particle duality, probability density, and the uncertainty principle.

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"I think I can safely say that nobody understands quantum mechanics."

— Richard P. Feynman

Introduction

Classical Physics: The Crown Jewel of the 19th Century

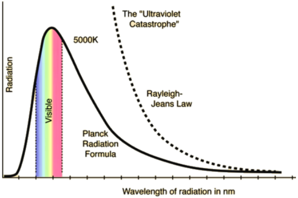
- Newton: Explained why apples fall and planets orbit — motion felt completely understood.
- Maxwell: Revealed that light, electricity, and magnetism are all waves dancing to the same rules.
- Everyday wonders: machines, engines, bridges, clocks — all worked perfectly according to classical laws.
- Scientists of the 1800s thought: "Physics is essentially done. The universe obeys predictable, beautiful laws."
- . . . until nature surprised everyone.

Why Care About Quantum Mechanics

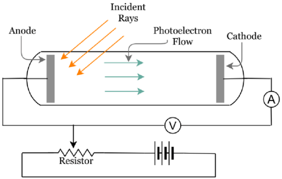
"Did you know the same rules that make atoms glow also power quantum computers that can solve problems classical computers cannot?"

- **Predicts the world accurately at small scales**
  Explains experiments where classical physics fails (atoms, photons, electrons).
- **Opens doors to new technologies**
  Lasers, semiconductors, MRI machines, quantum computers.
- **Explains phenomena classical physics cannot**
  Atomic stability, discrete spectral lines, photoelectric effect.
- **Shows the universe is stranger — and cooler — than imagined**
  Particles can behave like waves, exist in multiple states (superposition), and link across distances (entanglement).
- **Gives a head start in future technologies**
  Quantum technology is growing fast — understanding QM now prepares you for the future.

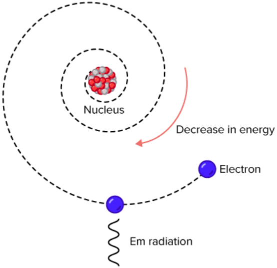
Where Classical Physics Failed — Experiments That Needed Quantum



Blackbody Radiation
Classical physics predicted infinite energy at high frequencies.
Planck solved it: *Energy comes in discrete packets (quanta).*



Photoelectric Effect
Brighter light did not always eject electrons.
Einstein: *Light behaves like particles (photons).*



Atomic Structure
Classical physics could not explain why atoms are stable.
Quantum theory: *Electrons occupy quantized energy levels (orbitals).*

What Does “Quantum” Mean?

Quantum = Tiny Discrete Chunks of Nature

- Some quantities in nature cannot take arbitrary values — they come in **discrete steps (quanta)**.
- **Electric Charge:**

  $$Q = Ne$$

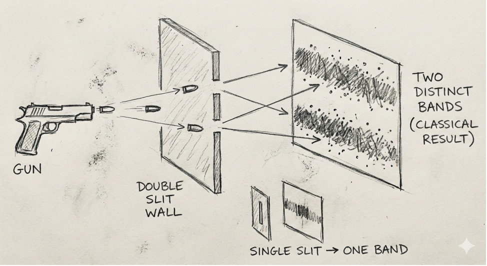
  where N is an integer, and e is the charge of one electron.
  Allowed: $2e$, $977e$, $3127 \times 10^{16}e$.
  Not allowed: $2.12e$, $34.76e$.
- **Atomic Energy Levels:** Electrons in atoms occupy only **specific allowed energies**. When an electron jumps between levels, it emits a photon with energy equal to the difference: a *quantum jump*.

Takeaway: Nature at tiny scales is discrete. Understanding quantum mechanics reveals this hidden structure and powers technologies classical physics cannot explain.

Double Slit Experiment: Classical Particles

Step 1: Classical Particles (Bullets)

- Shoot bullets at a wall with two slits.
- Each bullet goes through one slit only.
- On the screen: two distinct bands appear.
- Single slit → one band.

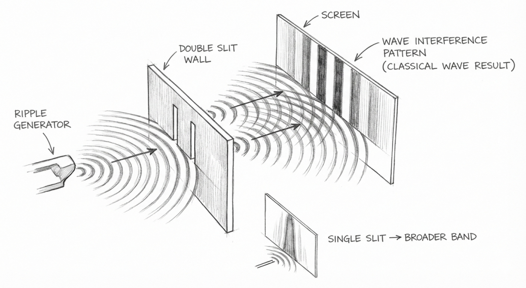


Observation: Classical particles behave intuitively.

Double Slit Experiment: Classical Waves

Step 2: Classical Waves (Water Waves)

- Pass waves (like water ripples) through two slits.
- Waves interfere → alternating peaks and troughs on the screen (interference pattern).
- Single slit → broader single band.

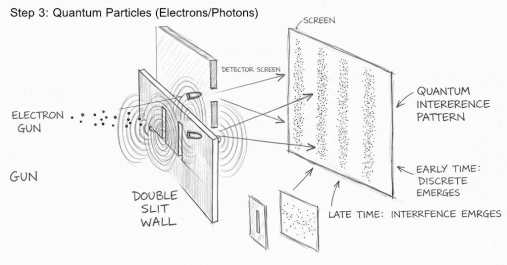


Observation: Waves produce interference, not just two bands.

Double Slit Experiment: Quantum Particles

Step 3: Quantum Particles (Electrons/Photons)

- Fire electrons or photons one at a time using an electron gun.
- Each particle hits the detector discretely, like bullets.
- Over time, an interference pattern emerges, just like waves.



Takeaway: Quantum particles act like bullets when measured, but like waves collectively. This is ***wave–particle duality****.*

Quantum Systems

In classical mechanics:

- A particle has a definite position x.
- A particle has a definite momentum p.
- Its motion is determined by Newton's laws.

However, microscopic systems (electrons, photons, atoms) do not behave according to classical expectations.

Need for a New Description

From the double-slit experiment:

- Particles arrive at the detector as discrete impacts.
- Yet an interference pattern emerges over time.

This behavior cannot be explained using classical particle mechanics.

Therefore, we require a new mathematical description.

The Quantum State

A quantum system is described by a function

$$\psi(x, t)$$

This function is called the **wavefunction**.

It contains all measurable information about the system.

Interpretation of the Wavefunction

The wavefunction itself is not directly observable.

The probability of finding a particle near position x at time t is

$$P(x, t) = |\psi(x, t)|^2$$

This is known as the Born Rule.

Superposition Principle

If ψ_1 and ψ_2 are valid quantum states,

then any linear combination

$$\psi = c_1\psi_1 + c_2\psi_2$$

is also a valid quantum state.

This property leads to interference effects.

Measurement in Quantum Mechanics

Before measurement, a system may exist in a superposition:

$$\psi = c_1\psi_1 + c_2\psi_2$$

Upon measurement, the system is found in one of the possible states.

The outcome is probabilistic, determined by $|c_1|^2$ and $|c_2|^2$.

Time Evolution of Quantum States

In classical mechanics, motion is governed by Newton's laws.

In quantum mechanics, the evolution of the wavefunction is governed by

$$i\hbar\frac{\partial\psi}{\partial t} = \hat{H}\psi$$

This is the Schrödinger equation.

Foundations of Quantum Mechanics

From our discussion, we observe that:

- A quantum system is described by a wavefunction.
- Physical predictions are probabilistic.
- Superposition is allowed.
- Measurement affects the system.
- Time evolution follows Schrödinger's equation.

These principles form the postulates of quantum mechanics.

Postulate 1: State of a Quantum System

The state of a quantum system is completely described by a normalized wavefunction

$$\psi(x,t)$$

The wavefunction must satisfy the normalization condition:

$$\int_{-\infty}^{\infty} |\psi(x,t)|^2 \, dx = 1$$

This ensures total probability equals 1.

Postulate 2: Observables and Operators

Every measurable physical quantity corresponds to a linear operator.

Examples:

Position:

$$\hat{x} = x$$

Momentum:

$$\hat{p} = -i\hbar \frac{d}{dx}$$

Measurement outcomes are eigenvalues of the operator.

Postulate 3: Measurement

Suppose we measure an observable represented by operator $\hat{A}$.

The possible outcomes of the measurement are the eigenvalues a satisfying

$$\hat{A}\,\phi_a(x) = a\,\phi_a(x)$$

If the system is in state $\psi(x)$,
the probability of obtaining a is determined by how much ψ overlaps with ϕ_a.

Postulate 4: Time Evolution

The time evolution of a quantum state is governed by

$$i\hbar\frac{\partial\psi}{\partial t} = \hat{H}\psi$$

where $\hat{H}$ is the Hamiltonian operator.

This is Schrödinger's equation.

Probability Density

The probability density is given by

$$\rho(x,t) = |\psi(x,t)|^2$$

Interpretation:

$$\rho(x,t)\,dx$$

is the probability of finding the particle between x and $x + dx$.

The wavefunction itself is not observable. Only $|\psi|^2$ has physical meaning.

Why Do We Need Operators?

In classical mechanics:

$$p = mv$$

is a number.

In quantum mechanics, the state is a function $\psi(x, t)$.

Physical quantities must act on the wavefunction.

Therefore, observables are represented by operators.

Position and Momentum Operators

Position operator:

$$\hat{x}\psi(x) = x\psi(x)$$

Momentum operator:

$$\hat{p}\psi(x) = -i\hbar\frac{d}{dx}\psi(x)$$

Momentum is related to spatial variation of the wavefunction.

Expectation Value

The expectation value of observable $\hat{A}$ is

$$\langle A \rangle = \int \psi^*(x)\, \hat{A}\, \psi(x)\, dx$$

This represents the average result of many measurements.

Does Order Matter?

In ordinary algebra:

$$ab = ba$$

Multiplication of numbers is commutative.

But in quantum mechanics, we deal with operators.

For operators $\hat{A}$ and $\hat{B}$:

$$\hat{A}\hat{B} \neq \hat{B}\hat{A}$$

in general.

The Commutator

The commutator of two operators is defined as

$$[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$$

If

$$[\hat{A}, \hat{B}] = 0$$

the operators commute.

If not, the order of application matters.

Physical Meaning of Commutation

If two observables commute:

$$[\hat{A}, \hat{B}] = 0$$

They share common eigenfunctions.

This means:

- They can be measured simultaneously.
- Both quantities can have definite values.

Non-Commuting Observables

If

$$[\hat{A}, \hat{B}] \neq 0$$

The operators do not share eigenfunctions.

This implies:

- Measuring one affects the other.
- They cannot both have precise values simultaneously.

Position and Momentum

Position operator:

$$\hat{x} = x$$

Momentum operator:

$$\hat{p} = -i\hbar \frac{d}{dx}$$

Their commutator is

$$[\hat{x}, \hat{p}] = i\hbar$$

They do not commute.

Consequence of Non-Commutation

Since

$$[\hat{x}, \hat{p}] \neq 0$$

Position and momentum cannot both be sharply defined.

This leads to the uncertainty relation:

$$\Delta x \, \Delta p \geq \frac{\hbar}{2}$$

Anticommutator

The anticommutator is defined as

$$\{\hat{A}, \hat{B}\} = \hat{A}\hat{B} + \hat{B}\hat{A}$$

Anticommutators appear in:

- Spin systems
- Fermionic operators

Heisenberg Uncertainty Principle

Position and momentum operators do not commute:

$$[\hat{x}, \hat{p}] = i\hbar$$

This leads to the uncertainty relation:

$$\Delta x \, \Delta p \geq \frac{\hbar}{2}$$

The uncertainty is fundamental, not due to experimental limitations.